A Strategic Theory of Network Status

Brian Rogers

MEDS, Northwestern University

July 10, 2008

SIAM 2008 Mini-Symposium

Motivation

- Why study social networks?
- Many kinds of complex relationships
  - Reputation systems
  - Research collaborations
  - Friendships
  - Teamwork
- Strategic considerations shape the structure of relationships
- These relationships impact outcomes
  - Aggregate and individual output
  - Quantity of information
  - Variety of goods and services

Setting

- Individuals have intrinsic value
- Allocate resources to others
- Resulting connections generate value
- Study what structures are likely to form and analyze their properties

Model elements

- Players $N = \{1, \ldots, n\}$, n finite
- Intrinsic values $\alpha = \{\alpha_1, \ldots, \alpha_n\}$ ($\alpha_i > 0$)
- Linking budgets $\beta = \{\beta_1, \ldots, \beta_n\}$ ($0 < \beta_i < 1$)
- Strategies: Allocate linking budget across other $n-1$ players
  - $\phi_i = (\phi_{i1}, \ldots, \phi_{in})$, ($\phi_{ii} = 0$, $\sum_j \phi_{ij} \leq \beta_i$)
  - S_i denotes feasible allocations
  - Strategy profile $\Phi = [\phi_{ij}]$
- Strength of link ij is $f(\phi_{ij})$,
  - $f(0) = 0$, f
  - strictly increasing and strictly concave
  - $\lim_{x \to 0} f'(x) = \infty$

Utility: directional separation

- Links confer utility by allowing intrinsic value to be shared
- Interaction may benefit both parties; I examine extreme cases
- Separate benefit flow into directional components: Giving and Taking
  - Giving: ϕ_{ij} sends value from i to j
  - Taking: ϕ_{ij} sends value from j to i

Utility: directional separation

- Links confer utility by allowing intrinsic value to be shared
- Interaction may benefit both parties; I examine extreme cases
- Separate benefit flow into directional components: Giving and Taking
  - Giving: ϕ_{ij} sends value from i to j
  - Taking: ϕ_{ij} sends value from j to i

Main result:

- Under Giving: Equilibrium networks are typically inefficient
- Under Taking: Equilibrium networks are always efficient

Utility: network values

- Network value v_i (depends on Giving/Taking)
- Utility: $u_i = \alpha_i + v_i$
- Network value in the two cases:

  Giving: $v_i = \sum_j f(\phi_{ji})(\alpha_j + v_j)$

  Taking: $v_i = \sum_j f(\phi_{ij})(\alpha_j + v_j)$

Utility: implications

- Marginal value derived from another agent depends on
  - Strength of link
  - Other's intrinsic value (exogenous)
  - Other's network value (endogenous)
  - More value from "better" individuals
- Value from all paths is counted
  - Redundancy is valued
  - Feedback effects
  - Wide externalities

Utility: deriving utility functions

- Matrix of link strengths $f(\Phi)$
- $u = \alpha + f(\Phi)u$ (Taking)
- $u = (I - f(\Phi))^{-1}\alpha$
- Let $A = (I - f(\Phi))^{-1}$
- Taking: $u = A\alpha,$ Giving: $u = A'\alpha$

Utility: the matrix A

$$A = \sum_{p=0}^{\infty} f(\Phi)^p = I + f(\Phi) + f(\Phi)^2 + \cdots$$

- Valid when $|f(\Phi)| < 1$, requires joint condition on β and $f(\cdot)$
- $f(\Phi)^p$ computes weight of all length-p paths
- A aggregates effects from all paths in $f(\Phi)$

Network definitions

$f(\Phi)$ is an

- Equilibrium network if Φ constitutes a pure strategy Nash equilibrium of $(N, \{S_i\}, \{u_i\})$
- "Efficient" (utilitarian) network if $\sum_i u_i(\Phi) \geq \sum_i u_i(\Phi')$ for all feasible Φ'
- Interior network if $\phi_{ij} > 0$ for all $j \neq i$
- Empty network if $\phi_{ij} = 0$ for all $j \neq i$

Results: giving

Nash Networks under Giving

Proposition. Interior equilibria satisfy the conditions $\sum_j \phi_{ij} = \beta_i$ for all $i \in N$, and

$$f'(\phi_{ij})a_{ji} = f'(\phi_{ij'})a_{j'i}$$

for all distinct $i, j, j' \in N$.

(Recall: a_{ji} = total weight of all paths from j to i in $f(\Phi)$)

Nash Networks under Giving

Proposition. Interior equilibria satisfy the conditions $\sum_j \phi_{ij} = \beta_i$ for all $i \in N$, and

$$f'(\phi_{ij})a_{ji} = f'(\phi_{ij'})a_{j'i}$$

for all distinct $i, j, j' \in N$.

(Recall: $a_{ji} =$ total weight of all paths from j to i in $f(\Phi)$)

- Empty network is always an equilibrium
- Non-interior: partitioned into interior subgroups
  - Eliminated by most refinements

Efficient Networks under Giving

Proposition. Any efficient network is interior, satisfies the conditions $\sum_j \phi_{ij} = \beta_i$ for all $i \in N$, and

$$f'(\phi_{ij}) \sum_k a_{jk} = f'(\phi_{ij'}) \sum_k a_{j'k}$$

for all distinct $i, j, j' \in N$.

Results: intrinsic values

Corollary: Under Giving, the equilibrium and efficient networks are independent of intrinsic values (α).

- “Good” strategies depend only on the network structure (Φ)

Results: giving

Theorem. Assume $n \geq 3$. There is an efficient Nash network under giving if and only if $\beta_i = \beta_j$ for all i, j.

- With homogeneous budgets, the regular network is both Nash and efficient
- With different budgets, the FOC for efficiency can not be satisfied in equilibrium

Results: taking

Theorem. Under Taking, Nash networks and socially efficient networks exist and are interior. They satisfy the conditions $\sum_j \phi_{ij} = \beta_i$ for all $i \in N$, and

$$f'(\phi_{ij})u_j = f'(\phi_{ij'})u_{j'}$$

for all distinct $i, j, j' \in N$.

Results: other linking technologies (f)

- $f(x) = x$
  - Similar message for efficiency of equilibria
- $f'(0) < \infty$
  - Allows analysis of component structures
- f non-increasing
  - May not be individually optimal to exhaust budget
  - This will break the efficiency result under Taking

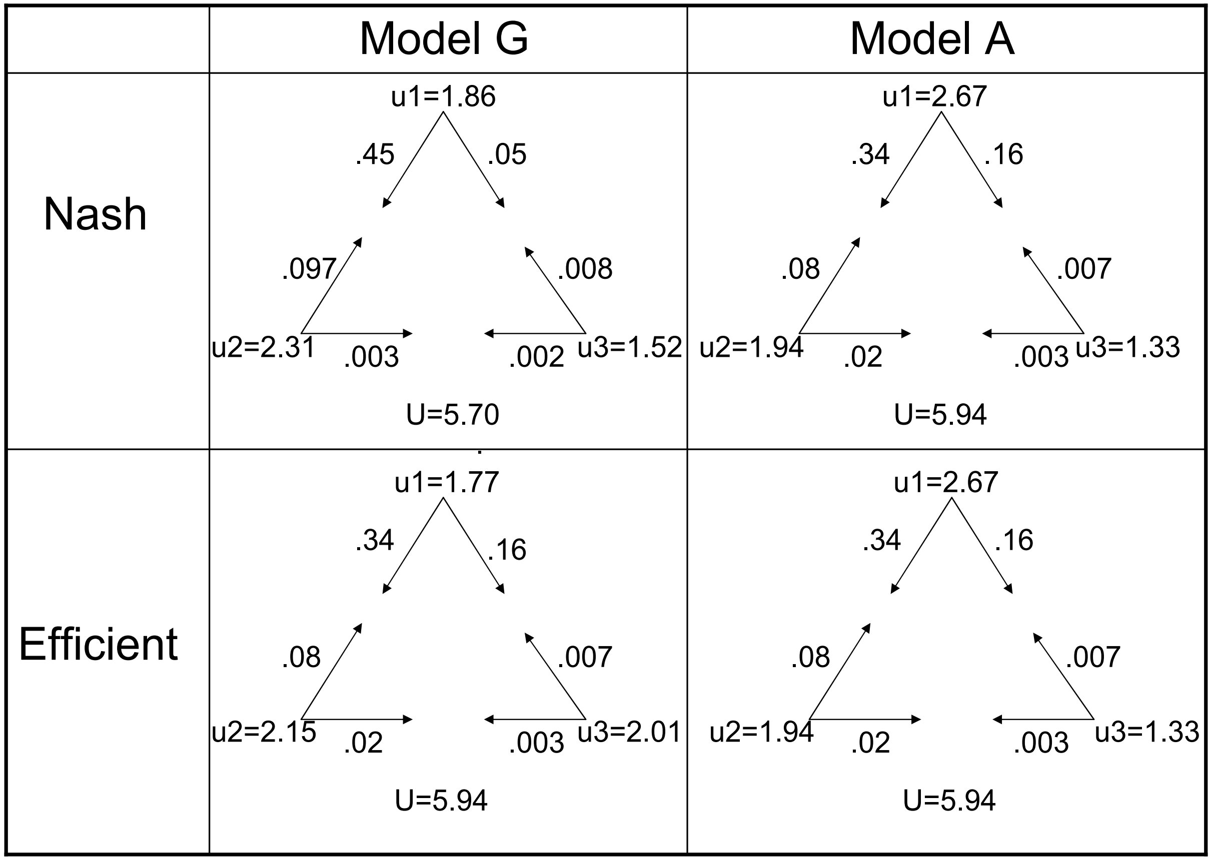
A few connections to the literature

- Strategic network formation
  - Strategic linking choices
  - Restrictive assumptions
  - (Jackson & Wolinsky (1996), Bala & Goyal (2000), Ballester, Calvó-Armengol & Zenou (2005))
- Interdependent utilities
  - Links interpreted as parameters in utility functions
  - Takes these patterns as given
  - (Bergstrom (1999), Bramoullé (2001), Hori (1997), Shinotsuka (2003))
- Sociology: centrality
  - Calculate centrality/prestige from a given network
  - Weight contributions by the value of the contributor
  - (Hubbell (1965), Bonacich (1972, 1987, 2005), Katz (1953))

Conclusion and further work

- New model of strategic networking
- Relationship strength is continuous
- Separate benefit flow into directional components
- Taking behavior is efficient, Giving typically is not
- Tie underlying heterogeneity of individuals to kinds of network structures that are likely to form
- Ties to “centrality” in sociology

Equilibrium and efficient networks



Results: intrinsic values

Corollary: Under Giving, the equilibrium and efficient networks are independent of intrinsic values.

- “Good” strategies depend only on the network structure

Network structures: symmetry

Asymmetric setup with symmetric prediction:

- Taking can also produce the regular network with asymmetric parameters
- Example: $\alpha = (3, 2, 2), \quad \beta = (0.015, 0.1, 0.1), \quad f(x) = \sqrt{x}$
  - Being well-connected can compensate for low intrinsic value

Symmetric setup with an Asymmetric prediction

- Under Giving, the regular network may not be the only equilibrium
- Example: $n = 3, \;\; \beta = (.1, .1, .1), \;\; f(x) = \delta x^{\lambda}, \;\; \lambda \approx 1$
  - Resembles a “star”

Results: intrinsic values

Comparing Taking and Giving under Homogeneous intrinsic qualities

- When $\alpha_i = \bar{\alpha}$ for all $i \in N$, the efficient networks in Model A and Model G coincide.
- Aggregate utility is the same across models at the efficient solution, but the distribution can be very different.

Network structures: heterogeneity

- Stars
  - Common in two-way flow models, not one-way
  - Robust prediction in this setting
- Taking: Single agent with larger intrinsic value or linking budget (or both)
- Giving: Single agent with larger linking budget

  Also in symmetric environments
- Stars are always efficient under Taking and never so under Giving

Network structures: heterogeneity

- “Standard” network models: wheel structure (Bala and Goyal (2000))
- Not predicted in this model
  - Decay
  - Wrong kind of heterogeneity
- Empty network
  - Occurs in binary link models for high costs
  - Approximated here by small budgets
  - Equilibrium under Giving

Results: linear case

- Constant returns to investment: $f(x) = x$

  Proposition. Under Giving with identical budgets, the efficient networks are those for which $\sum_j \phi_{ij} = \beta_i$ for all i.

- There are both efficient and inefficient equilibria.
  - Empty network
  - Regular network

Results: linear case

Proposition. Under Giving with strictly ordered budgets:

- All paired networks are equilibria
- The unique efficient network is assortatively paired

Results: linear case

Proposition. Under Taking with identical budgets and intrinsic values:

- Equilibrium and efficient networks coincide
- They are those for which $\sum_j \phi_{ij} = \beta$ for all i

Results: other forms

- $f'(0) < \infty$
- f non-increasing
- f non-concave

Conclusion

- New model of strategic networking
- Relationship strength is continuous
- Benefit calculation produces well-known centrality measure
- Separate benefit flow into directional components
- Tie underlying heterogeneity of individuals to kinds of network structures that are likely to form
- Taking behavior is efficient, Giving typically is not
- Future work
  - Two-way flow
  - Experiments

Centrality

- Sociologists have been concerned with measuring centrality
- Many ideas:
  - Degree
  - Closeness
  - Betweenness
  - Eccentricity
- Weighted centrality
  - Katz (1953)
  - Hubbell (1965)
  - Bonacich (1972, 1987, 2005)