Power law and exponential decay of inter contact times between mobile devices

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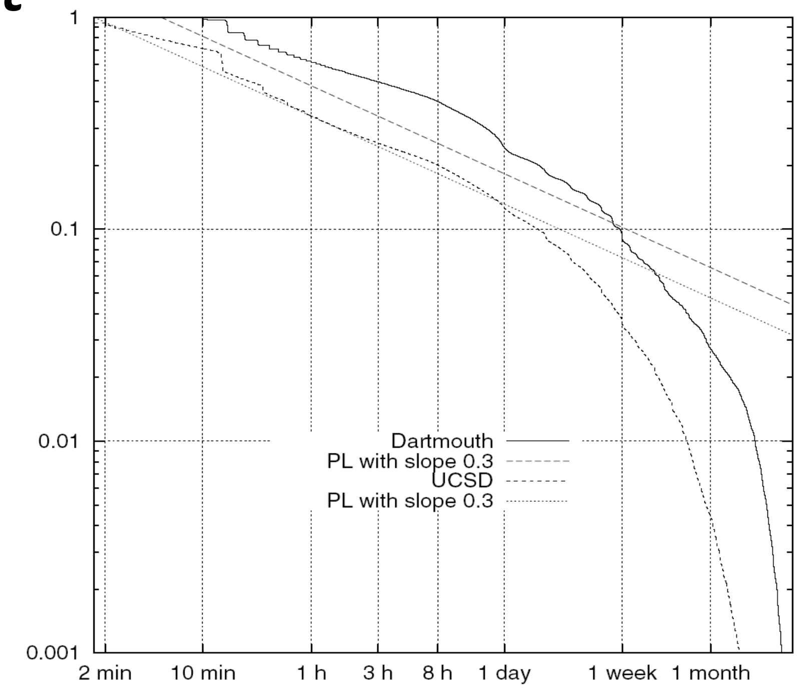
Opportunistic communications

Power-law finding

- *Distribution of inter-contact time exhibit **power-law** over a large range!*
  - Chaintreau et al. -- Infocom 06



- *State of the art until 2006:*
  - *Distribution of inter-contact time between mobile devices decays **exponentially***

Power tail hypothesis

- Hypothesis based on empirical finding
  - ***Power-law tail***
- Bad news for forwarding schemes!
  - For sufficiently heavy tail, expected packet delay is **infinite** for any packet forwarding scheme

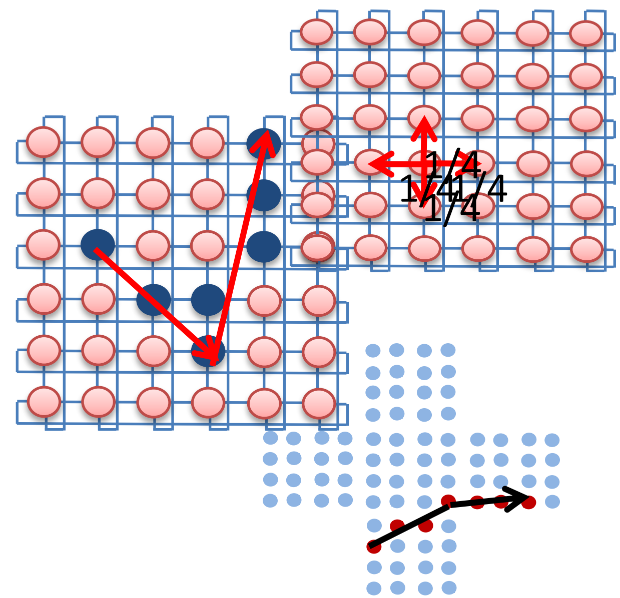
Assume a Pareto CCDF of inter-contact time :

$$F^0(t) = \left(\frac{t_0}{t}\right)^\alpha \qquad \alpha > 0, t \geq t_0 > 0$$

If a <= 1, expected packet forwarding delay infinite for any forwarding scheme

Failure of mobility models

- Empirical finding:
  - Power-law decay



- But:

Mobility models feature exponential decay!

Contributions

- Empirical evidence: We find a ***dichotomy*** in the inter-contact time distribution
  - **Power-law up to a point (order half a day), exponential decay beyond**
  - In sharp contrast to the power-law tail hypothesis
- Analytical results
  - Dichotomy **supported** by (simple) mobility models
  - Exponential tail applicable to a broad class of models
- Understanding the origins of the dichotomy
  - Can return time explain the inter-contact time dichotomy?
  - Is dichotomy an artifact of aggregation?

Outline

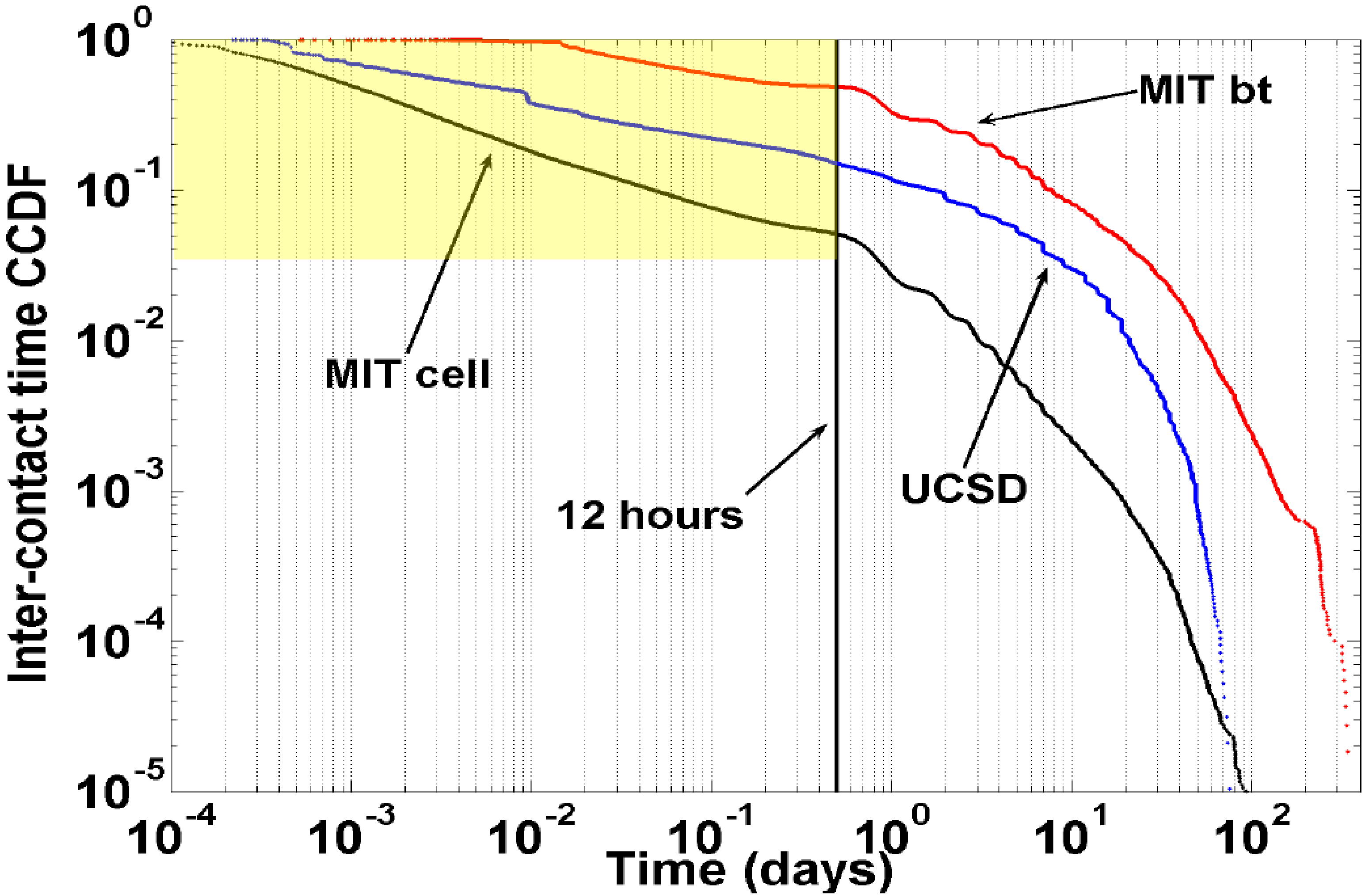
- Power-law, exponential dichotomy
- Mobility models support the dichotomy
- Origins of the dichotomy
- Conclusion

Datasets

Name	Technology	Duration	Devices	Contacts	Mean Inter-contact Time	Year
UCSD	WiFi	77 days	275	116,383	24 hours	2002
Vehicular	GPS	6 months	196	9,588	20.8 hours	2004
MITcell	GSM	16 months	89	1,891,024	3.5 hours	2004
MITbt	Bluetooth	16 months	89	114,046	87 hours	2004
Cambridge	Bluetooth	11.5 days	36	21,203	14 hours	2005
Infocom	Bluetooth	3 days	41	28,216	3.3 hours	2005

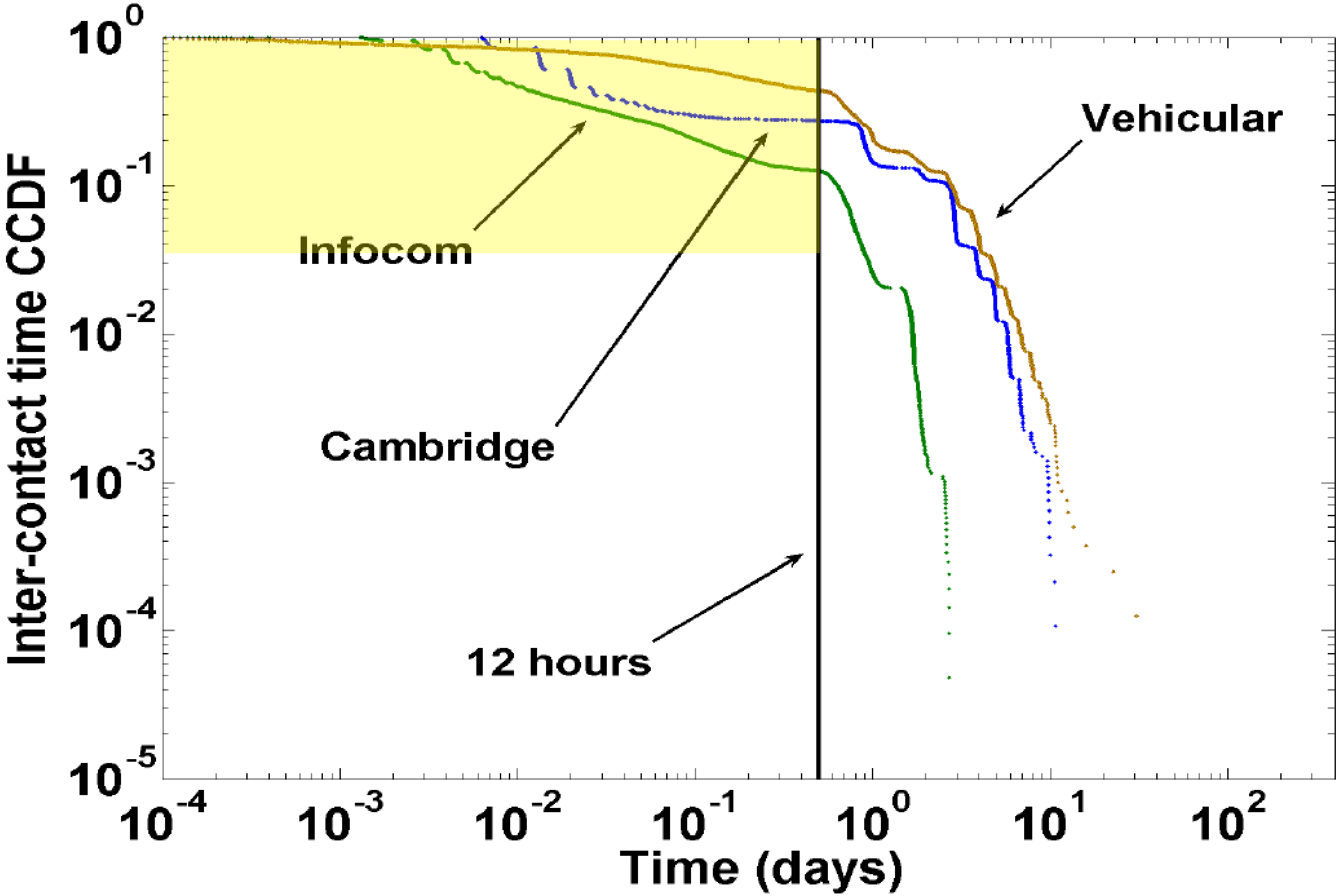
- All but the vehicular dataset are public and were used in earlier studies
- Vehicular is a private trace (thanks to Eric Horvitz and John Krumm, Microsoft Research MSMLS project)

Power law



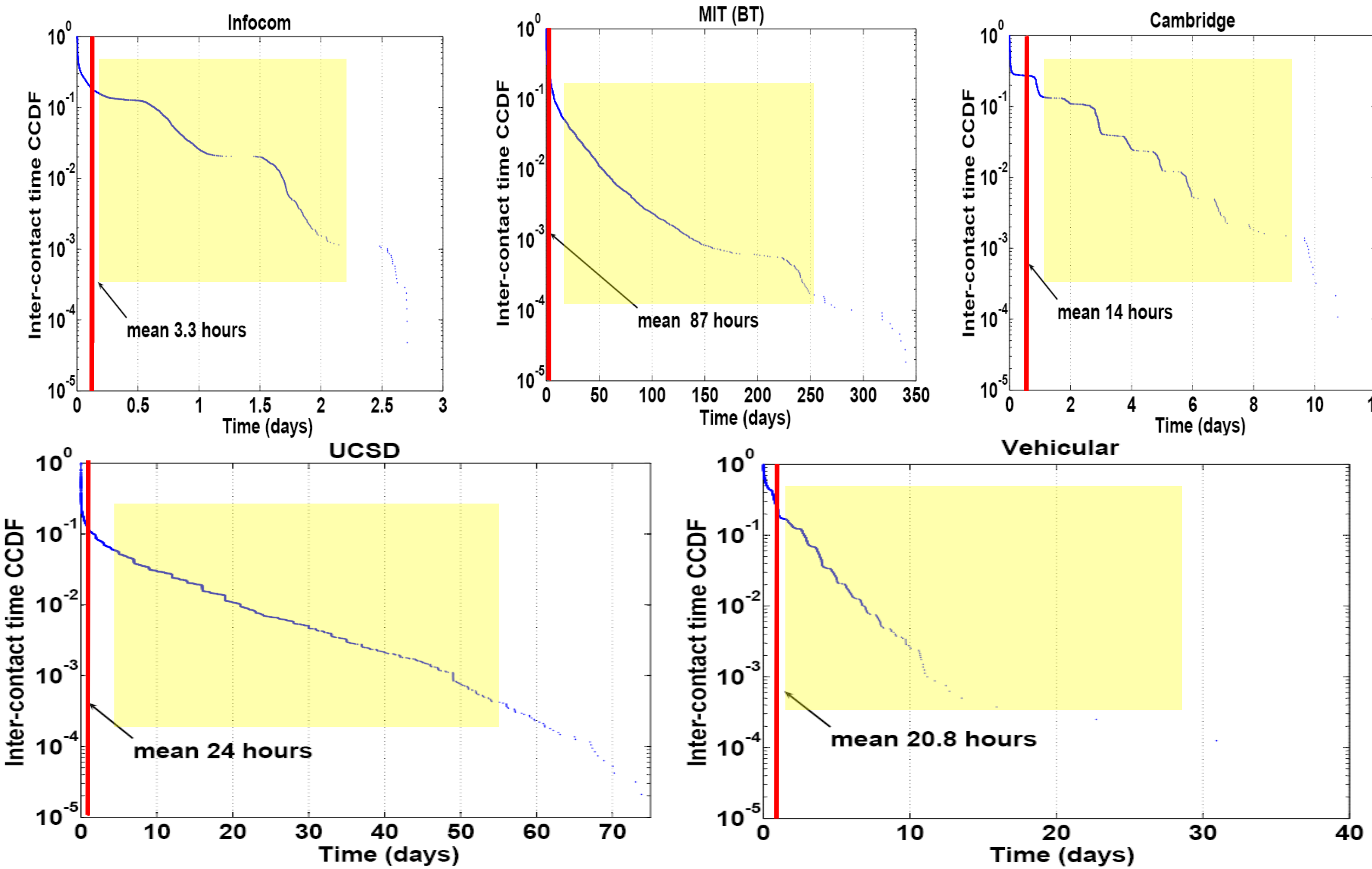
Inter-contact time CCDF
Time (days)
MIT bt
MIT cell
UCSD
12 hours
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2

Power law (2)



Inter-contact time CCDF
Time (days)
Vehicular
Infocom
Cambridge
12 hours

Exponential decay



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Inter-contact time is exponentially bounded

RETURN TIME FOR FINITE MARKOV CHAIN

Let X_n be an irreducible Markov chain on some finite state space S and let Δ be a subset of S ($\Delta \neq \emptyset$ and $\Delta \neq S$). Let R be the return time to Δ. The stationary distribution of R is such that

$$\mathbb{P}(R > n) \sim \varphi(n)e^{-\beta n}, \ \textit{large } n$$

where $\varphi(n)$ is a trigonometric polynomial and $\beta > 0$.

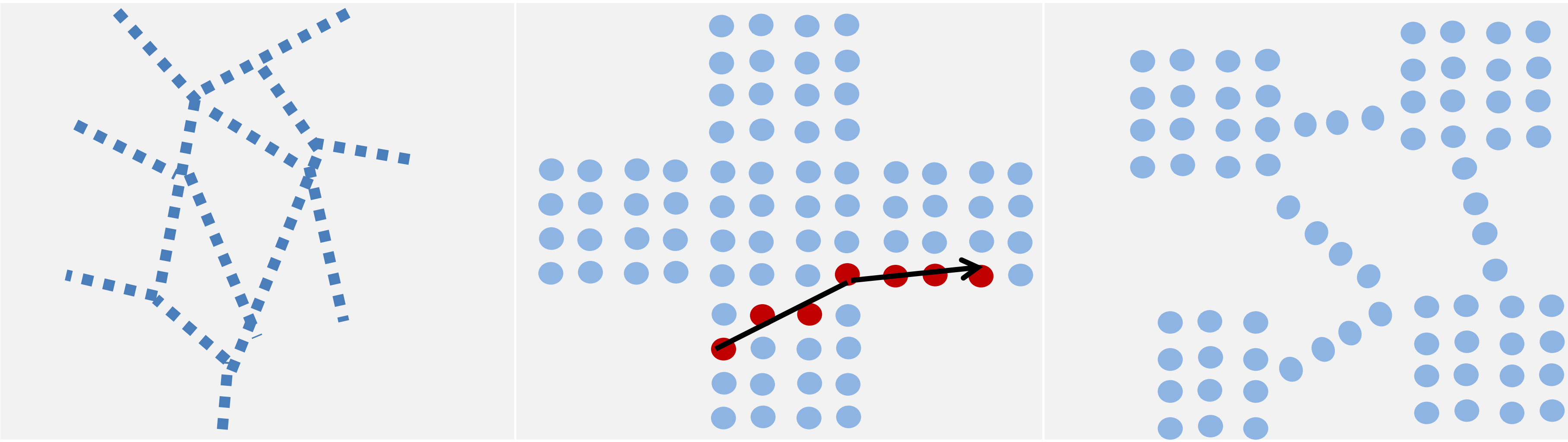
$$\varphi(n) = \sum_{k=1}^{K}[a_k \cos(\omega_k n) + b_k \sin(\omega_k n)]$$

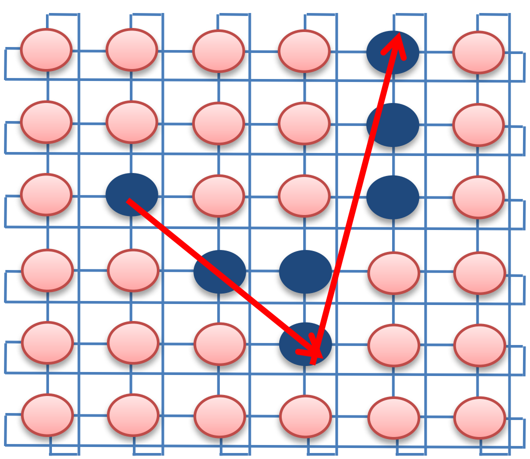
f(n) ~ g(n) means f(n)/g(n) goes to 1 as n goes to infty

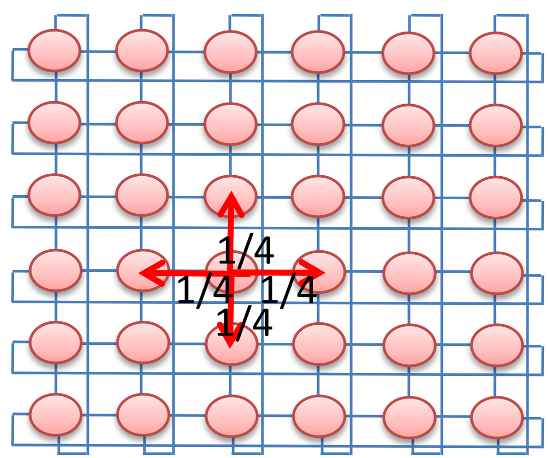
What does this mean?

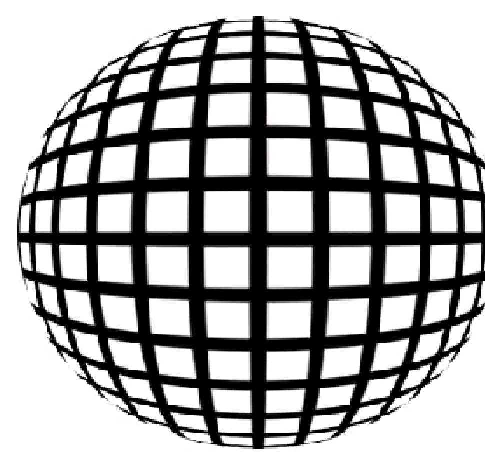
- Inter-contact time is exponentially bounded:
  - if the mobility of two nodes is described by an irreducible Markov chain on a finite state space
- General result for a broad class of models
  - No need for further assumptions
  - Enough that the chain is irreducible

Examples of applicable mobility models

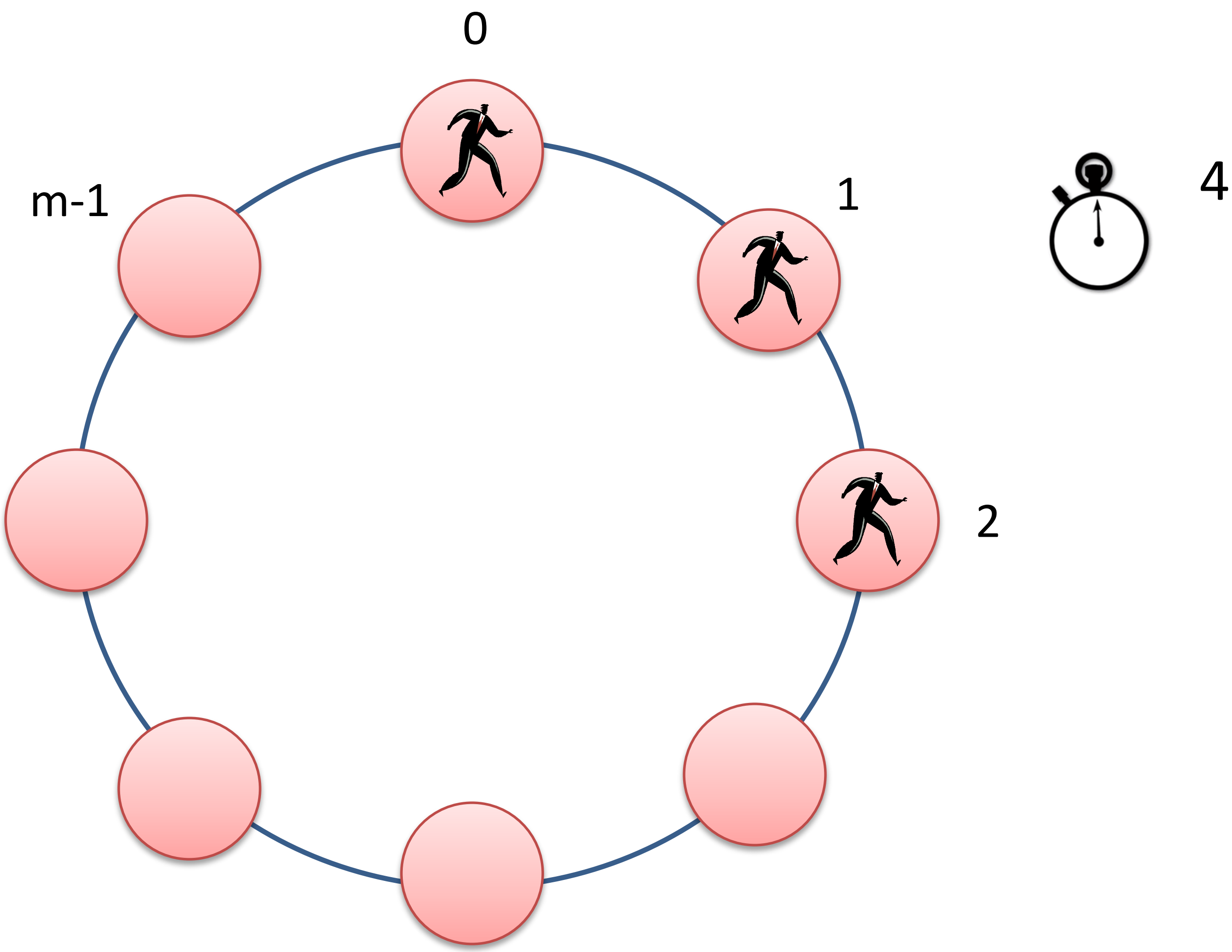




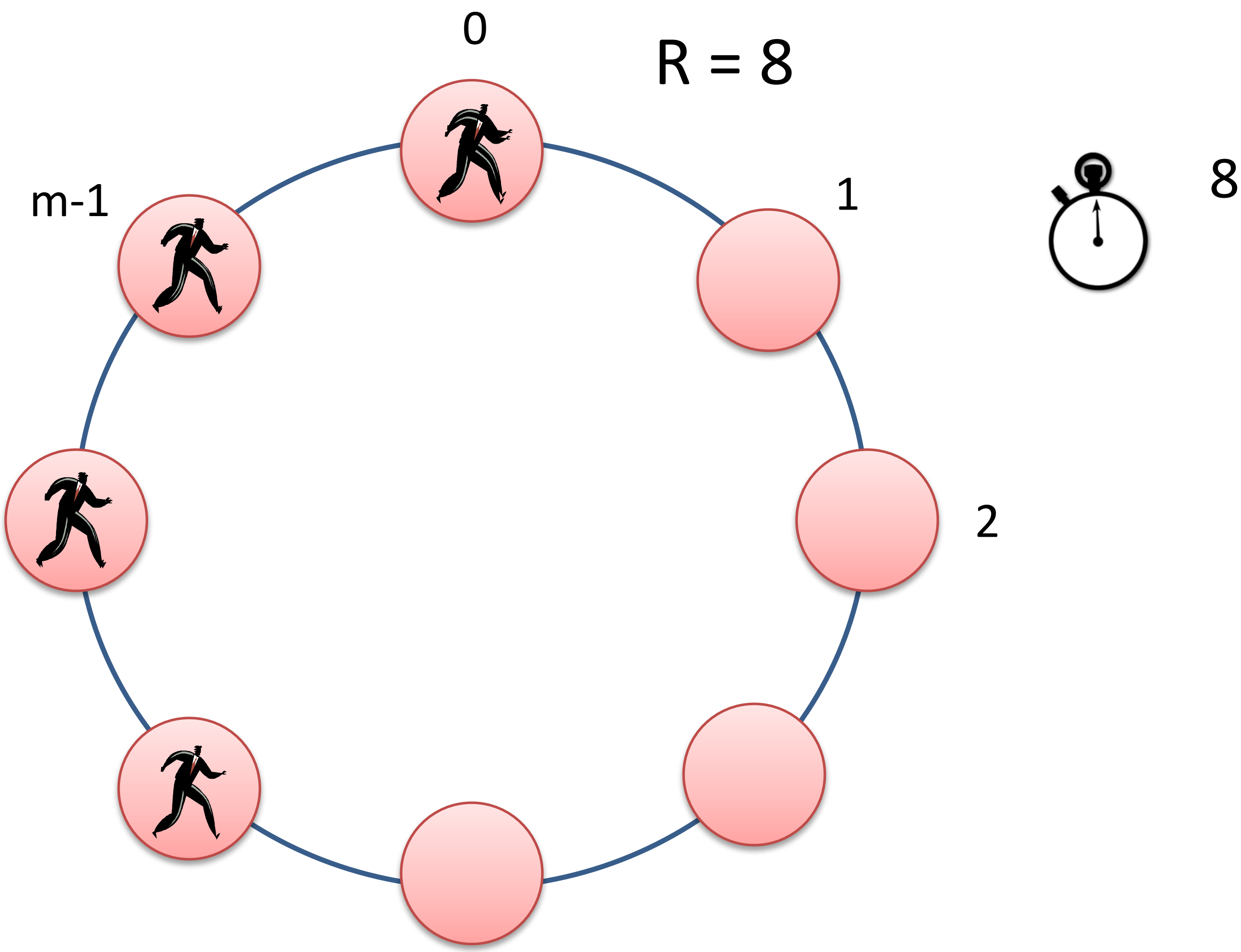




Simple random walk on a circuit



Return time to a site



Return time to a site of a circuit

- Expected return time:

$$\mathrm{E}(R) = m$$

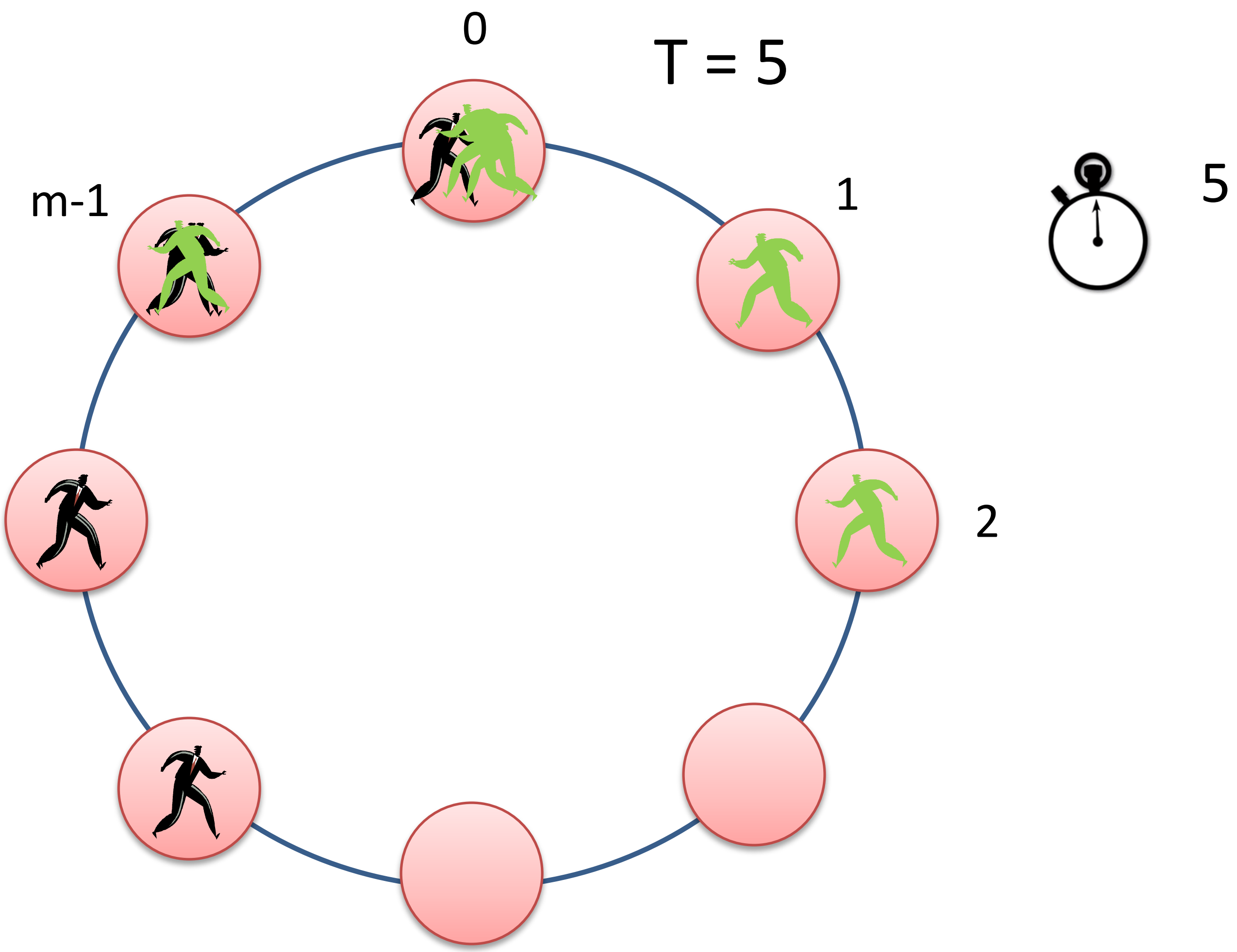
- Power-law for infinite circuit:

$$\mathrm{P}(R > n) \sim \sqrt{\frac{2}{\pi}} \frac{1}{n^{1/2}}, \text{large } n$$

- Exponentially decaying tail:

$$\mathrm{P}(R > n) \sim \varphi(n) e^{-\beta n}, \text{large } n, \beta > 0$$

Inter-contact time



Inter-contact time on a circuit

- Expected inter-contact time:

$$\mathrm{E}(T) = m - 1$$

- Power-law for infinite circuit:

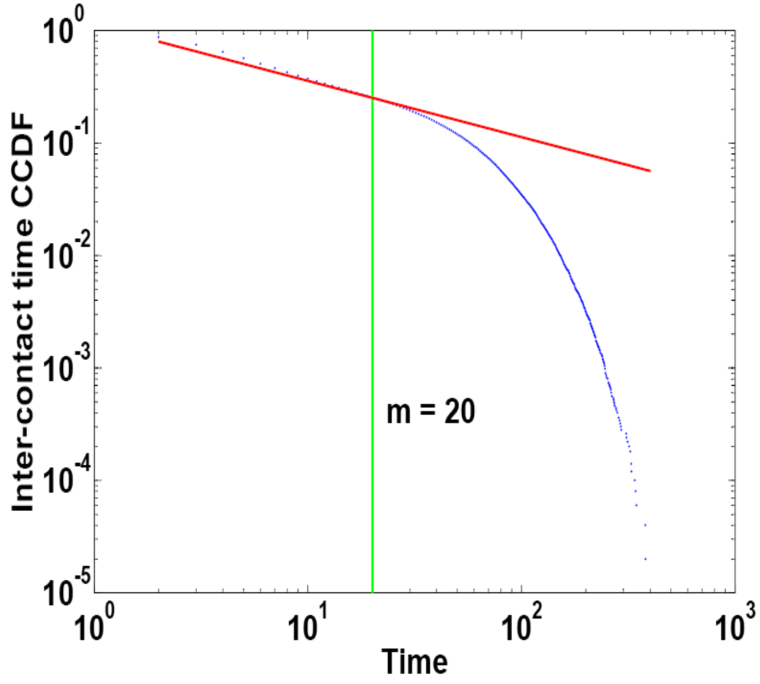
$$\mathrm{P}(T > n) \sim \frac{2}{\sqrt{\pi}} \frac{1}{n^{1/2}}, \text{ large } n$$

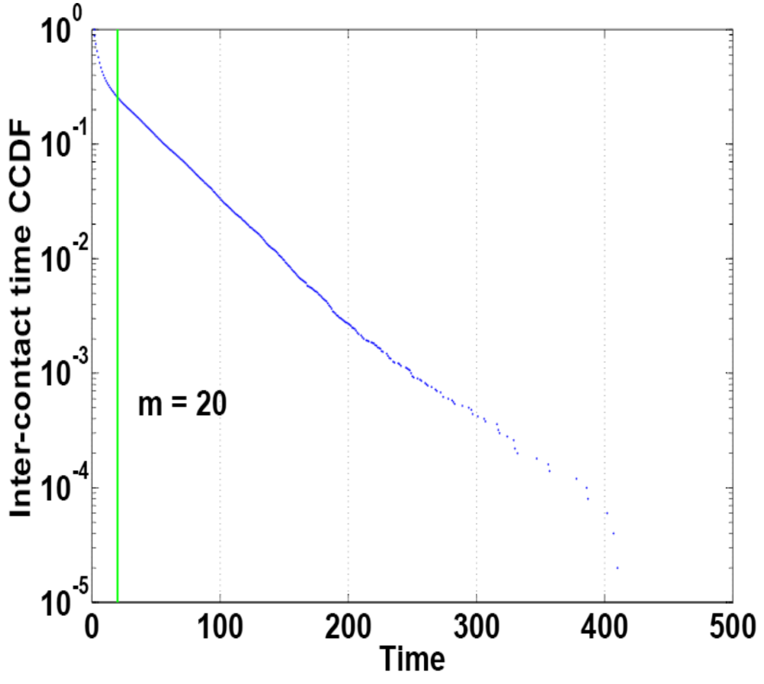
- Exponentially decaying tail:

$$\mathrm{P}(T > n) \sim \varphi(n) e^{-\beta n}, \text{ large } n, \beta > 0$$

Qualitatively same as return time to a site

Inter-contact time on a circuit of 20 sites





- Power-law, exponential dichotomy

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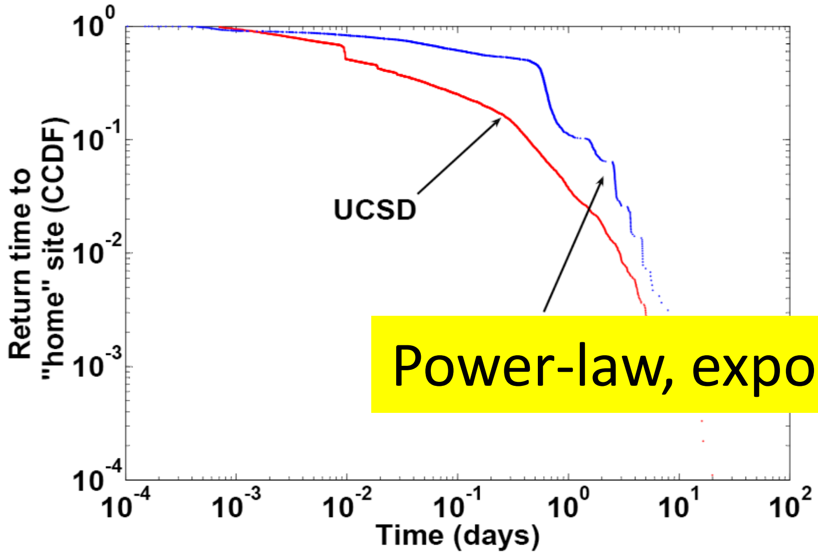
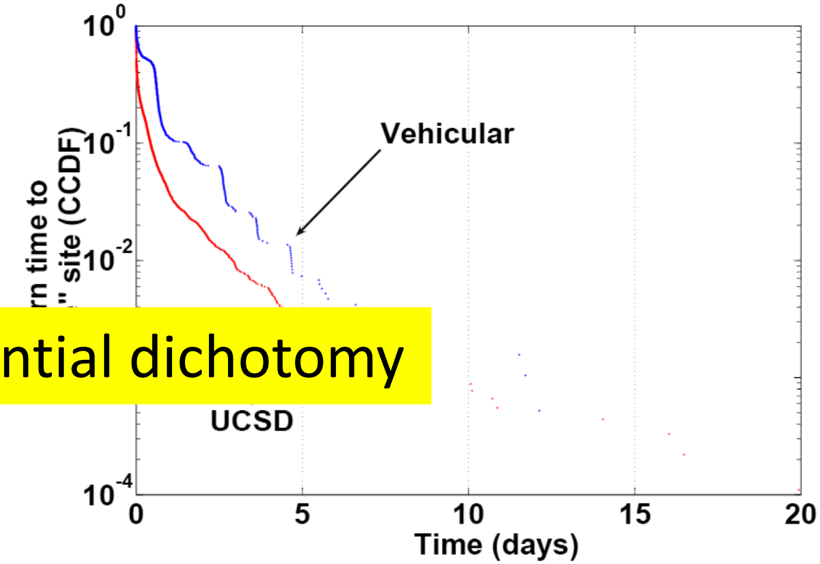
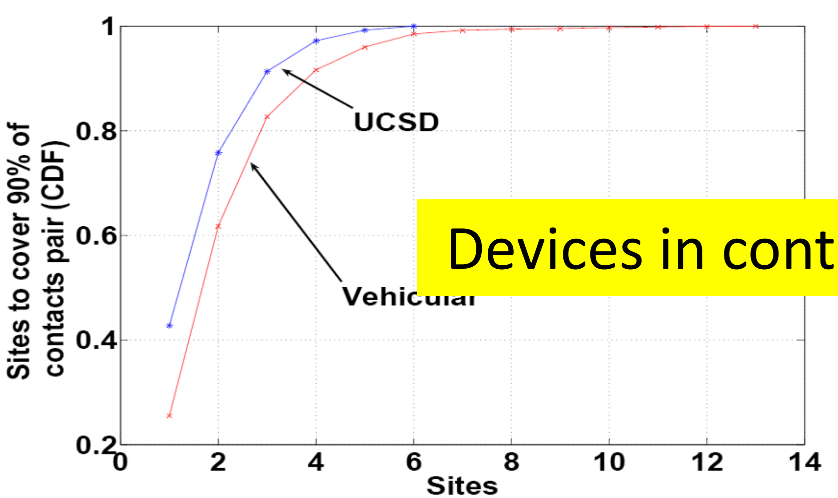
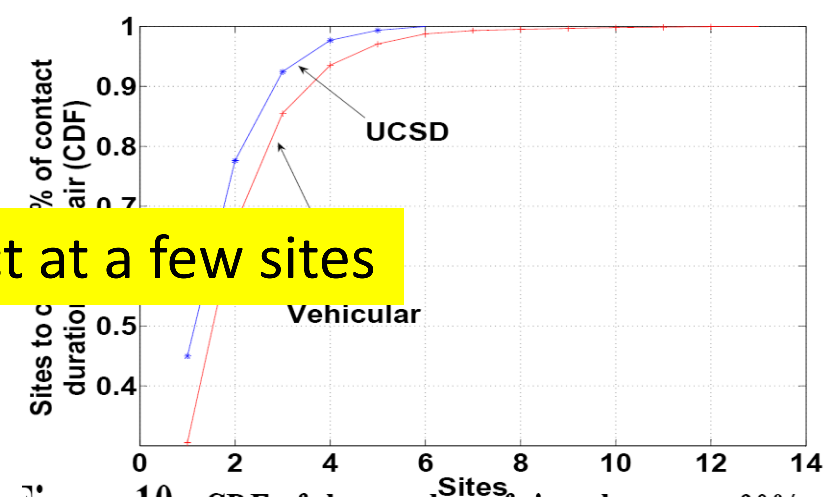
Is inter-contact time distribution explained by return time?





Power-law, exponential dichotomy





Devices in contact at a few sites

Aggregate viewpoint

- In most studies: Inter-contact time CCDF estimated
  - over a time interval
  - taking samples over all device pairs
- Unbiased estimate if inter contacts for distinct device pairs **statistically identical**
- But, behavior is not homogeneous across devices
  - Is power-law an artifact of aggregation?

Aggregate viewpoint

- CCDF of all pair inter-contact times equivalent to:
  - Picking a time t uniformly at random
  - Picking a device pair p uniformly at random
  - Observe the inter-contact time for pair p from time t

- Aggregate vs. device pair viewpoint:
  - In general not the same
  - Some variability across device-pairs
  - Dichotomy is also present for distinct device-pairs

Summary & Implications

- Dichotomy in the distribution of inter-contact time
  - Power-law up to a characteristic time
  - Exponential decay beyond
  - ➢ **Infinite packet delay does not appear relevant**
- Mobility models
  - Simple models support the observed dichotomy
  - Exponential tail for a broad class of models
  - ➢ **Should not be abandoned as unrealistic**
- Origins of dichotomy
  - Return time might explain dichotomy inter-contact time
  - Heterogeneity does not appear to be the cause

First ACM SIGCOMM Workshop on Social Networks (WOSN 2008)

WOSN 2008 — The First ACM SIGCOMM Workshop on Online Social Networks

The organizing committee is delighted to invite you to WOSN 2008, co-located with ACM SIGCOMM 2008 in Seattle, WA, USA.

Call for Papers

With half a billion active users, online social networks (OSN) have attained critical mass and triggered intense research interest in collaborative systems and the analysis of the structure and properties of online communities.

WOSN will bring together researchers and practitioners to discuss the challenges and important questions posed by emerging online social applications. Of particular interest are problems related to network and system architecture design that can best support emerging and future social and collaborative systems, and how those social networks can shape the design of existing distributed systems and real networks. The goal of the workshop is to facilitate cross-disciplinary discussion of relevance to computer networking, involving novel ideas and applications, and experimental results.

The workshop solicits original, previously unpublished ideas on completed work, position papers, and/or work-in-progress papers. We encourage papers that propose new research directions or could generate lively debate at the workshop.

Topics

Topics of interest include, but are not limited to the following:

- Implications of social networking on network and distributed systems design
- System design for social networks
- Network architecture design to support large scale social applications
- Search strategies in social networks
- Rating, review, reputation, and trust systems
- Recommendation / collaborative filtering systems
- Expertise / interest tracking
- Anonymity and privacy
- Measurement and analysis of online communities
- Social media analysis: blogs and friendship networks
- Mobile social networks
- Information sharing and forwarding
- Decentralized (ad hoc) network applications and services
- Challenges posed by social networks