Modeling Social and Economic Exchange in Networks

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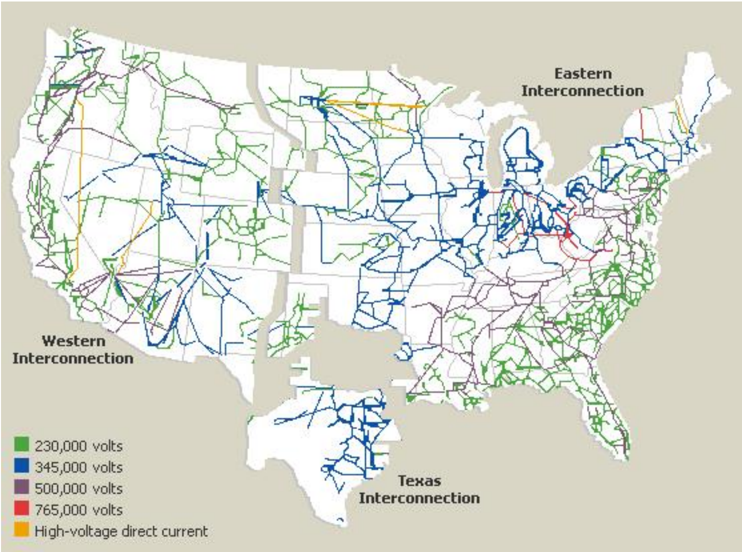
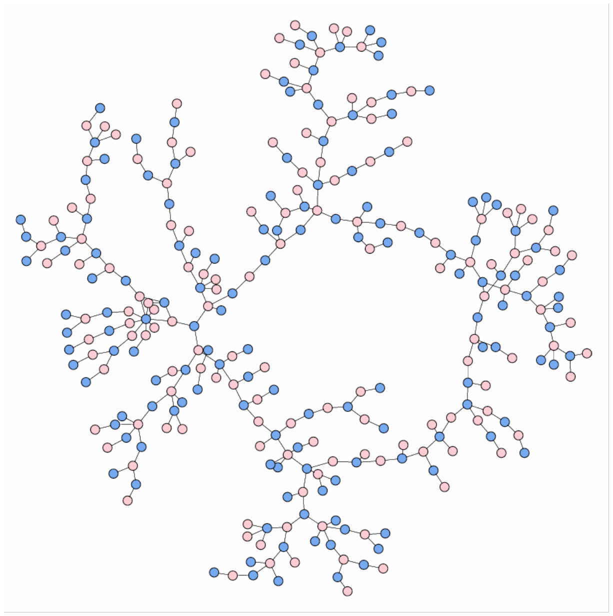
Joint work Éva Tardos (Cornell)

Networks Mediate Exchange



U.S. electric grid



High-school dating (Bearman-Moody-Stovel 2004)

Networks mediate exchange and power

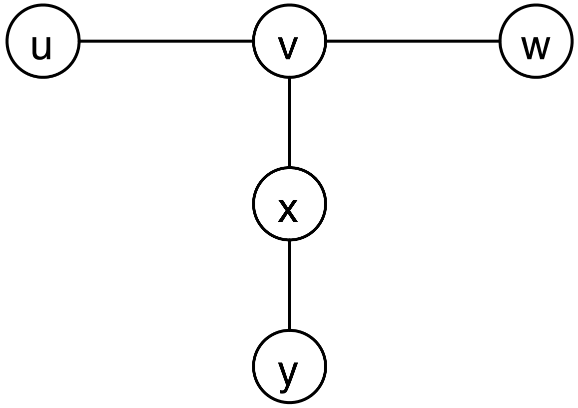
- Economic exchange: markets structured as networks.
- Social exchange [Emerson 1962, Blau 1964, Homans 1974]:
  - Social relations produce value that is divided unequally among the participants.

Network Exchange Theory

To what extent do social power imbalances have structural causes?

An widely-used experimental framework for social exchange.

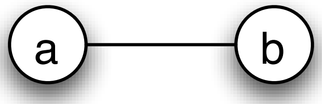
[Cook-Emerson 1978, Cook et al. 1983, Markovsky et al. 1988, Friedkin 1992, Bienenstock-Bonacich 1992, Cook-Yamagishi 1992, Skvoretz-Willer 1993, Willer 1999, ...]



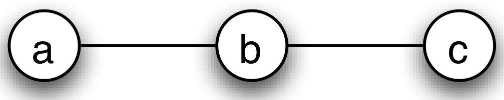
- A different human subject plays each node of the graph.
- A fixed amount of money (say $1) is placed on each edge.
- Nodes engage in free-form negotiation against a fixed time limit over how to split the money.
- Allowed to reach agreement with at most one neighbor.

Results robust against variations (what subjects can see, how they can communicate).

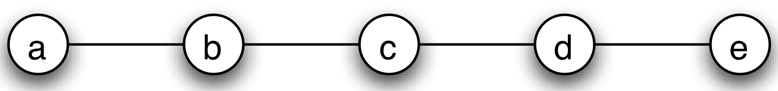
What happens?



Result: Even split.

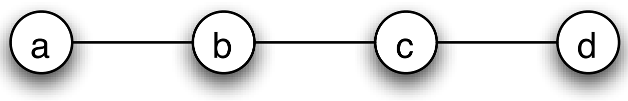


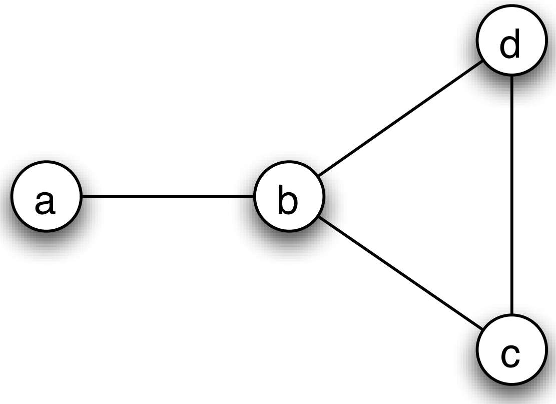
Result: Node *b* gets almost all the value in its one exchange.



Result: Nodes *b* and *d* get almost all the value.
Node *c*'s "centrality" is useless.

Weak Power



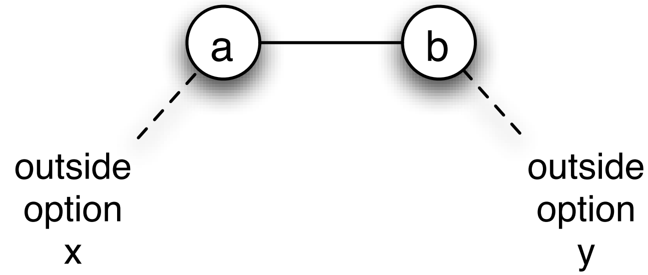


Some more subtle examples.

- 4-node path: Nodes *b* and *c* get roughly $\frac{7}{12}$-$\frac{2}{3}$ in practice.
  - *b* has the power to exclude *a*, but it is costly to exercise this power.
- "Stem graph:" Node *b* gets a bit more than in 4-node path, but still bounded away from 1.
  - *b*'s power to exclude *a* is a bit less costly to exercise.

Can we build a simple model to predict the outcomes of network exchange?

Modeling Outcomes: Pairwise Bargaining



Bargaining with outside options.

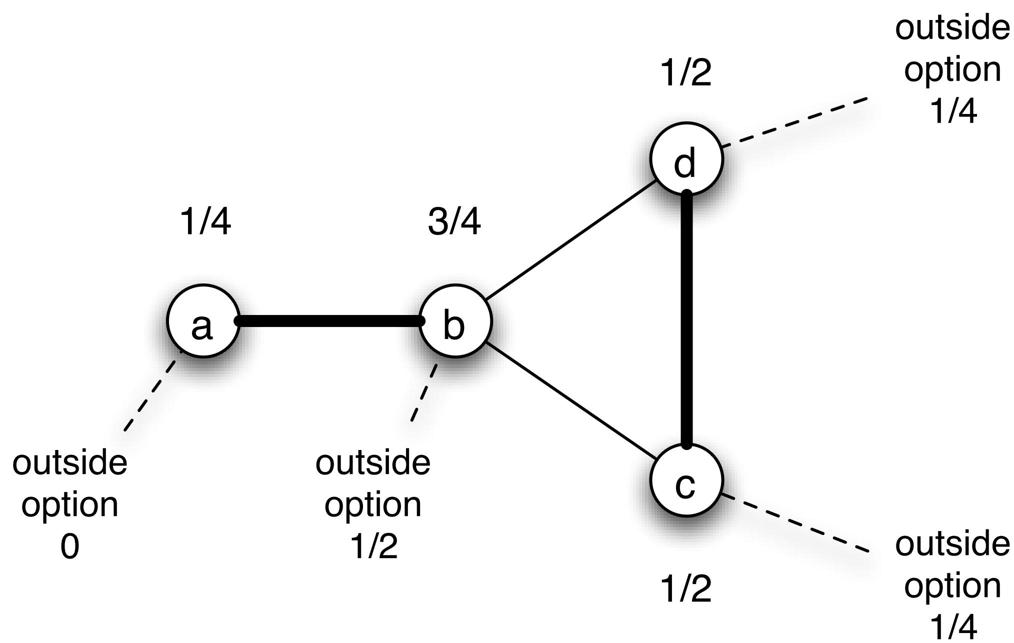
Suppose a has option of x and b has option of y if negotiations break down.

- Negotiation is really over the surplus $s = 1 - x - y$.
- Nash bargaining solution predicts nodes will split surplus evenly:

  $x + \frac{1}{2}s$ for node a, and $y + \frac{1}{2}s$ for node b.

Defining Balanced Outcomes

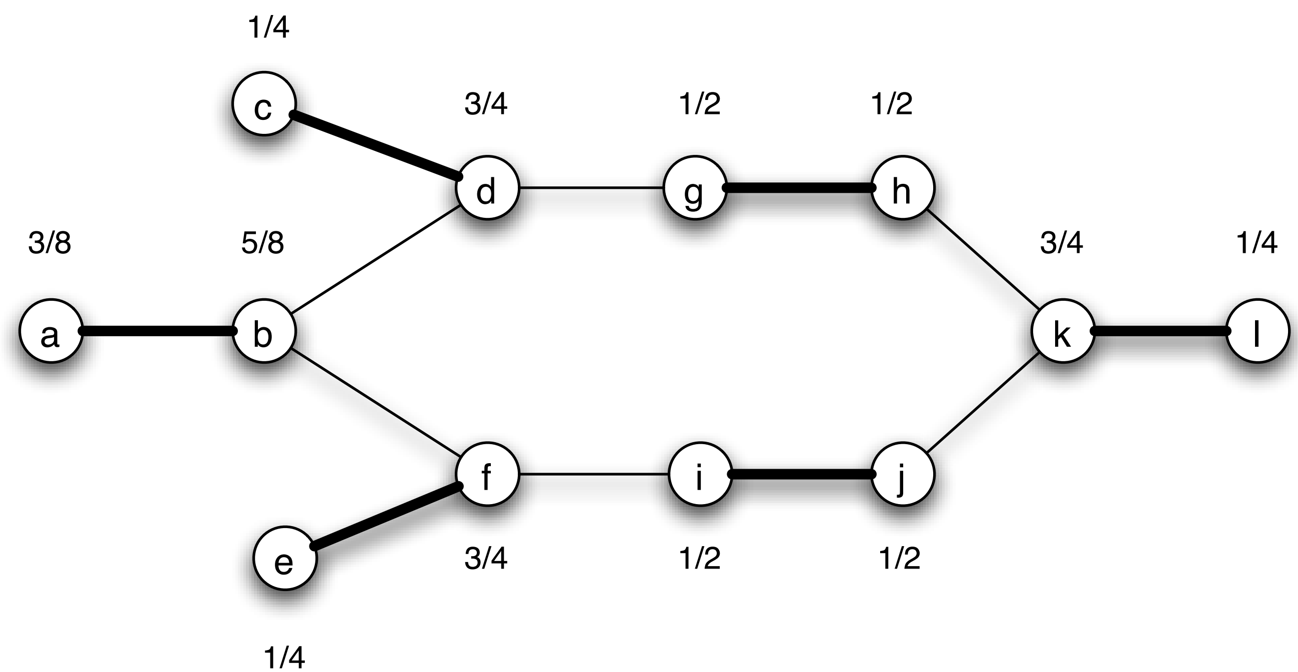
Modeling overall outcome: a matching M and a value for each node.



For each node, determine its best outside option, given the other edges in M.

- The outcome is balanced if the outcome on each $(v, w) \in M$ constitutes the Nash bargaining solution for v and w with respect to their best outside options [Cook-Yamagishi 1992].

Some Basic Questions

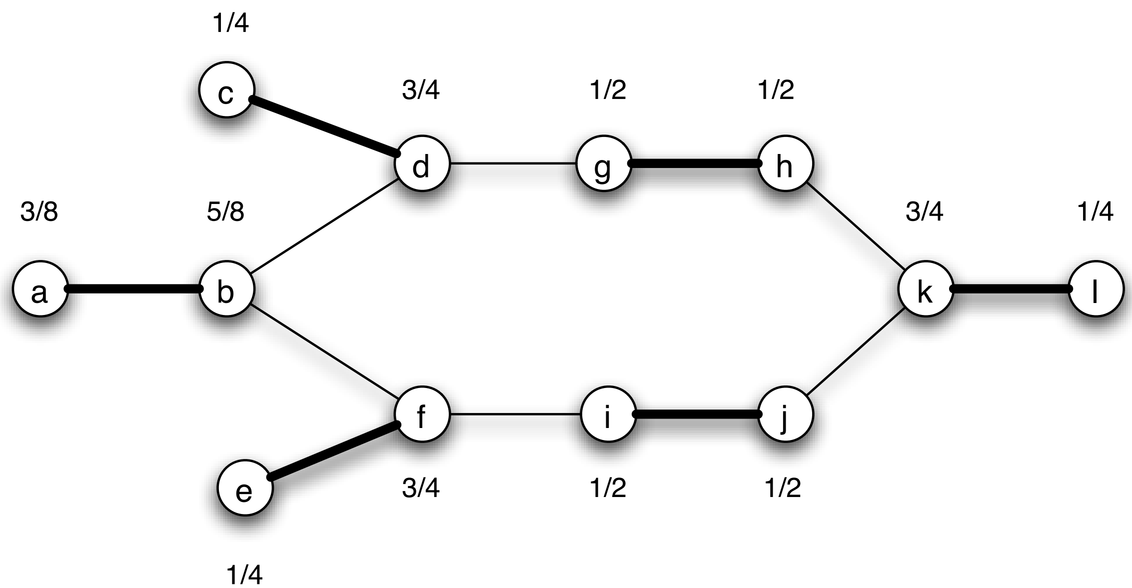


Can we

- characterize which graphs have balanced solutions?
- efficiently compute a balanced solution for a given G?
- efficiently represent the set of all balanced solutions for G?

Given the fixed-point nature of the definition, not a priori clear that balanced solutions should be rational/finitely-representable.

Main Results



Kleinberg-Tardos 2008:

- Characterize existence of balanced outcomes.
- Polynomial-time algorithm to compute a balanced outcome, and to build representation for set of all balanced outcomes.
- Results extend to edge-weighted graphs.

Balanced outcomes correspond to particular interior (or at least non-extreme) points in fractional relaxation of matching problem.

Further Related Work

Connections to several models of buyer-seller matching markets.

- The core of a matching market
  [Shapley-Shubik'72].
- Ascending auctions on bipartite graphs
  [Demange-Gale-Sotomayor'86; Kranton-Minehart'01]
- Algorithms to compute competitive equilibrum
  [Devanur et al '02; Kakade et al '04; Codenotti et al '04; Cole-Fleischer'07]
- Mechanism design on bipartite graphs
  [Leonard'83; Babaioff et al '05; Chu-Shen'06]
- Price-determination through bargaining on bipartite graphs
  [Calvó-Armengol'01, Charness et al '04, Corominas-Bosch'04, Navarro-Perea'01]

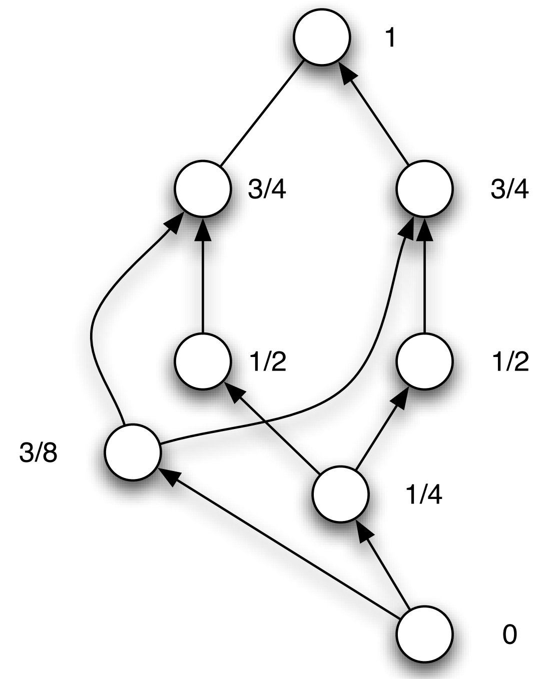
Most of these lines of work focus on outcomes corresponding to extreme points of the dual fractional matching problem.

A Combinatorial Problem on Posets

Given a poset $\mathcal{P}$ with min $\bot$, max $\top$.

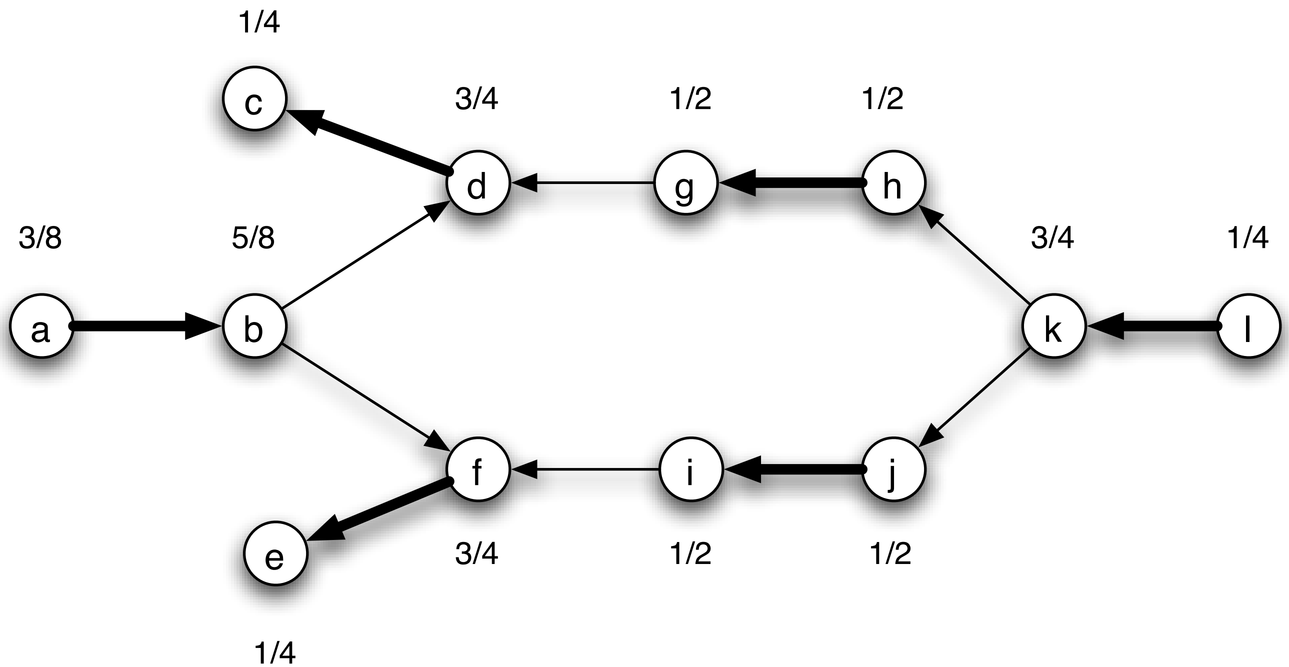
Consistent labeling: assignment of x_i to each $i \in \mathcal{P}$ s.t.

- $x_\bot = 0$,
- $x_\top = 1$, and
- $x_i \leq x_j$ when $i \preceq j$ (order polytope constraints)



- A balanced labeling is a consistent labeling such that that x_i is the midpoint of $\max_{j \preceq i} x_j$ and $\min_{i \preceq k} x_k$.
  - A combinatorially defined interior point of the order polytope.
- Theorem: Every poset has a unique balanced labeling.
  - Generalization: Given a consistent labeling of a subset of the elements, there is a unique extension to a labeling that is balanced on the remaining elements.

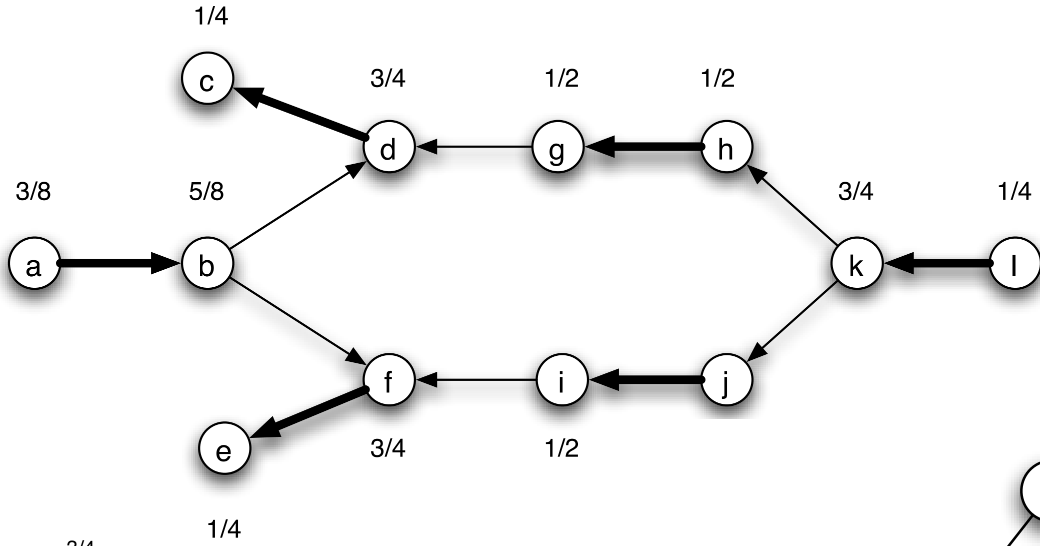
From Posets to Unique Perfect Matchings

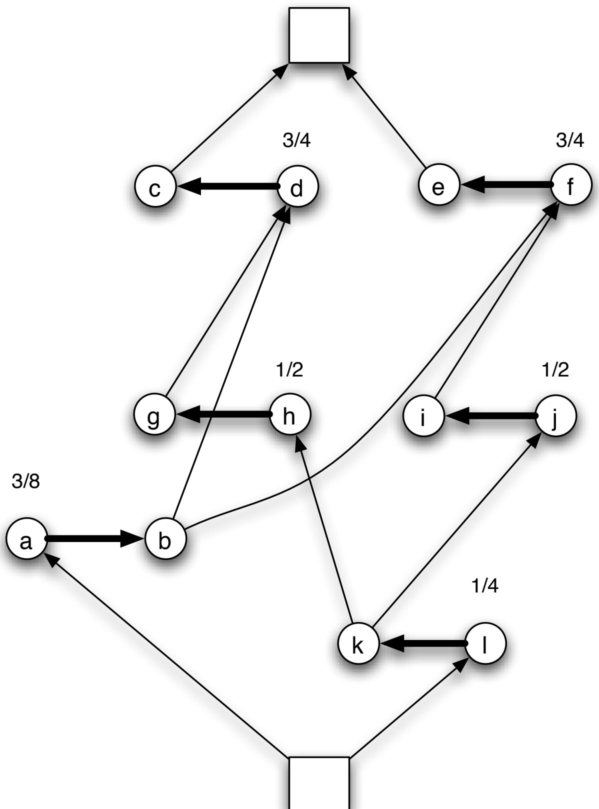


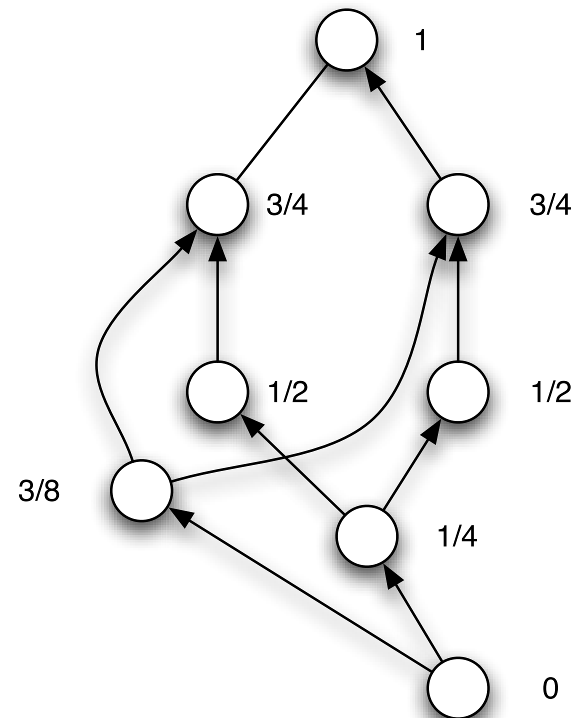
Let G be a bipartite graph with two sides X and Y, and a unique perfect matching.

- Direct all matching edges from X to Y, and all unmatched edges from Y to X.
- Resulting digraph is acyclic; defines poset on X via reachability.
- A balanced labeling of this poset gives balanced outcome in G.

From Posets to Unique Perfect Matchings

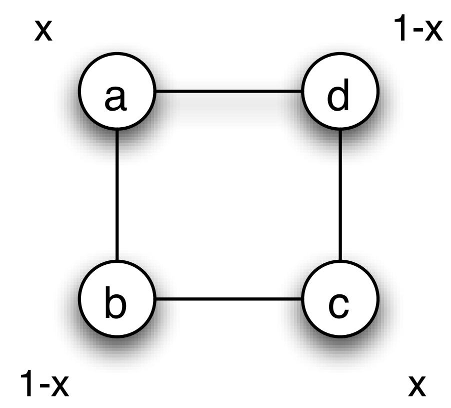






General Bipartite Graphs

Moving to general bipartite graphs.

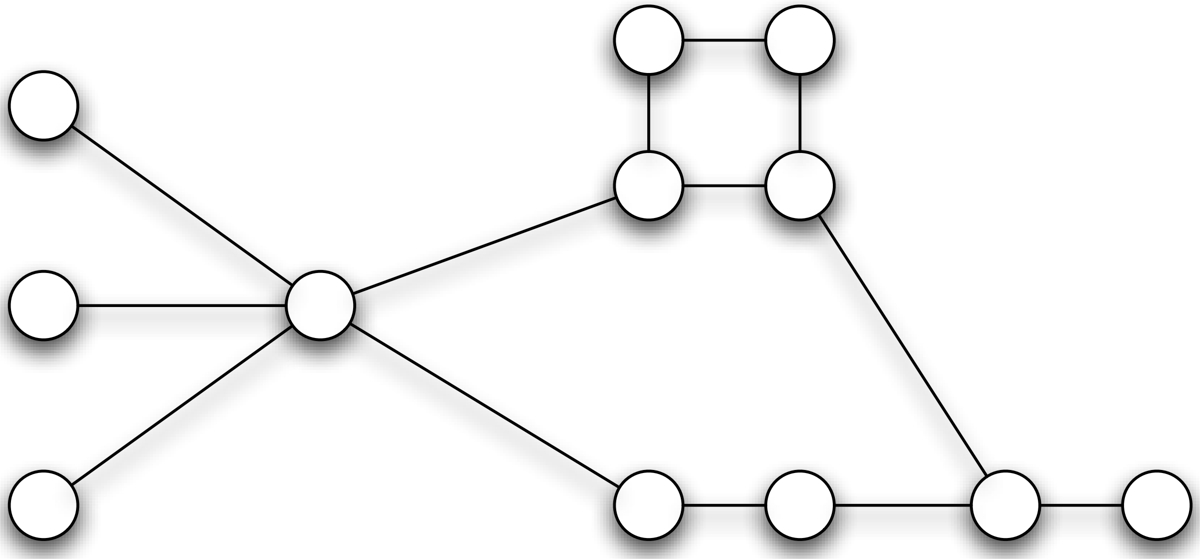


New phenomenon: self-supporting subgraphs.

- On an even cycle, can alternate values of x and $1 - x$ for any x, and it will be balanced.

This observation plus the poset problem are the key ingredients, via Edmonds-Gallai decomposition and elementary subgraph structure.

Edmonds-Gallai and Elementary Subgraphs



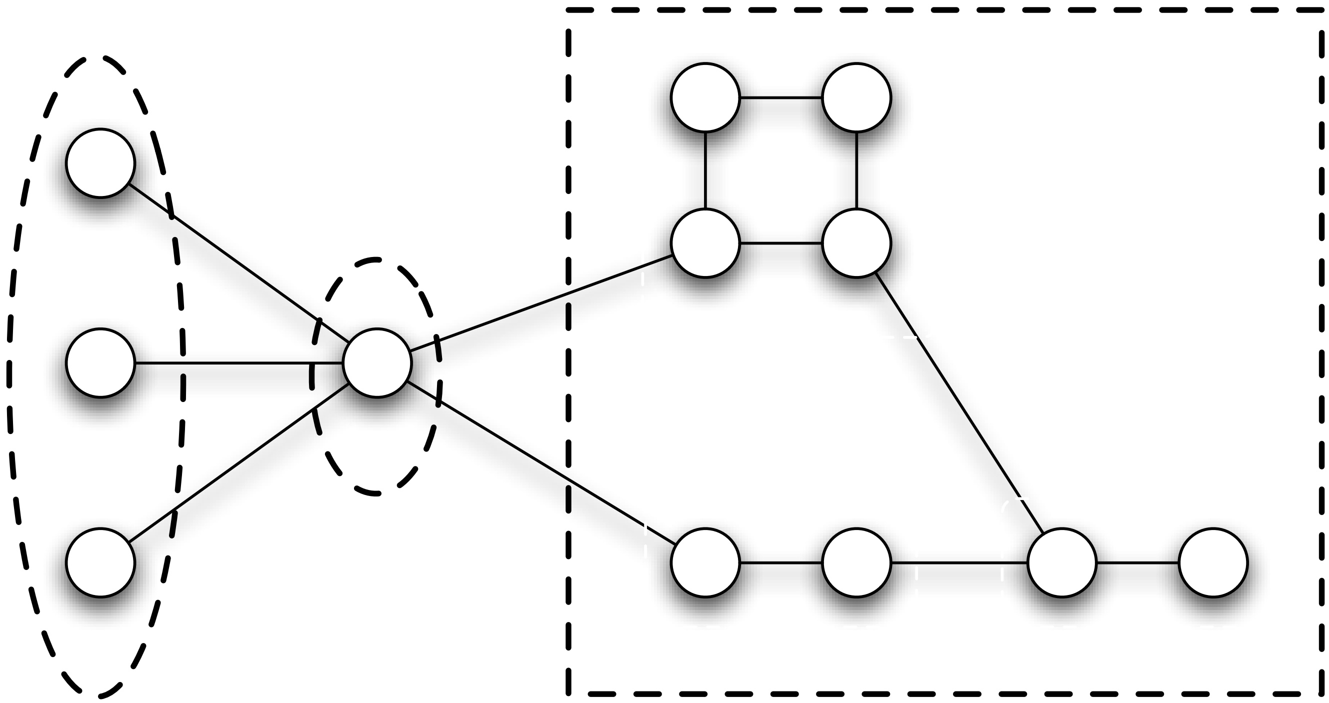
Edmonds-Gallai decomposition partitions a graph into three sets:

- $D =$ nodes not in every perfect matching.
- $A =$ nodes adjacent to D.
- $C =$ the remaining nodes, which have a perfect matching.

Further decompose C into elementary subgraphs:

components of subgraph on edges in some perfect matching.

Edmonds-Gallai and Elementary Subgraphs



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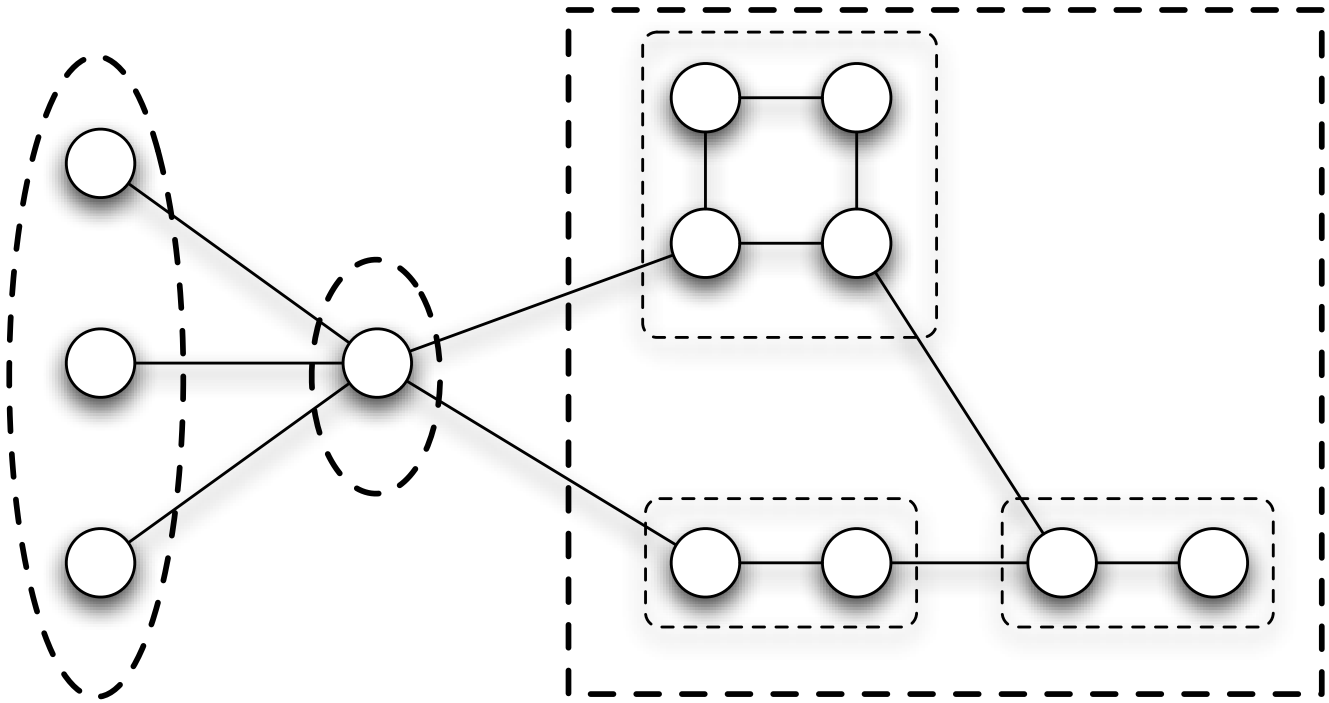

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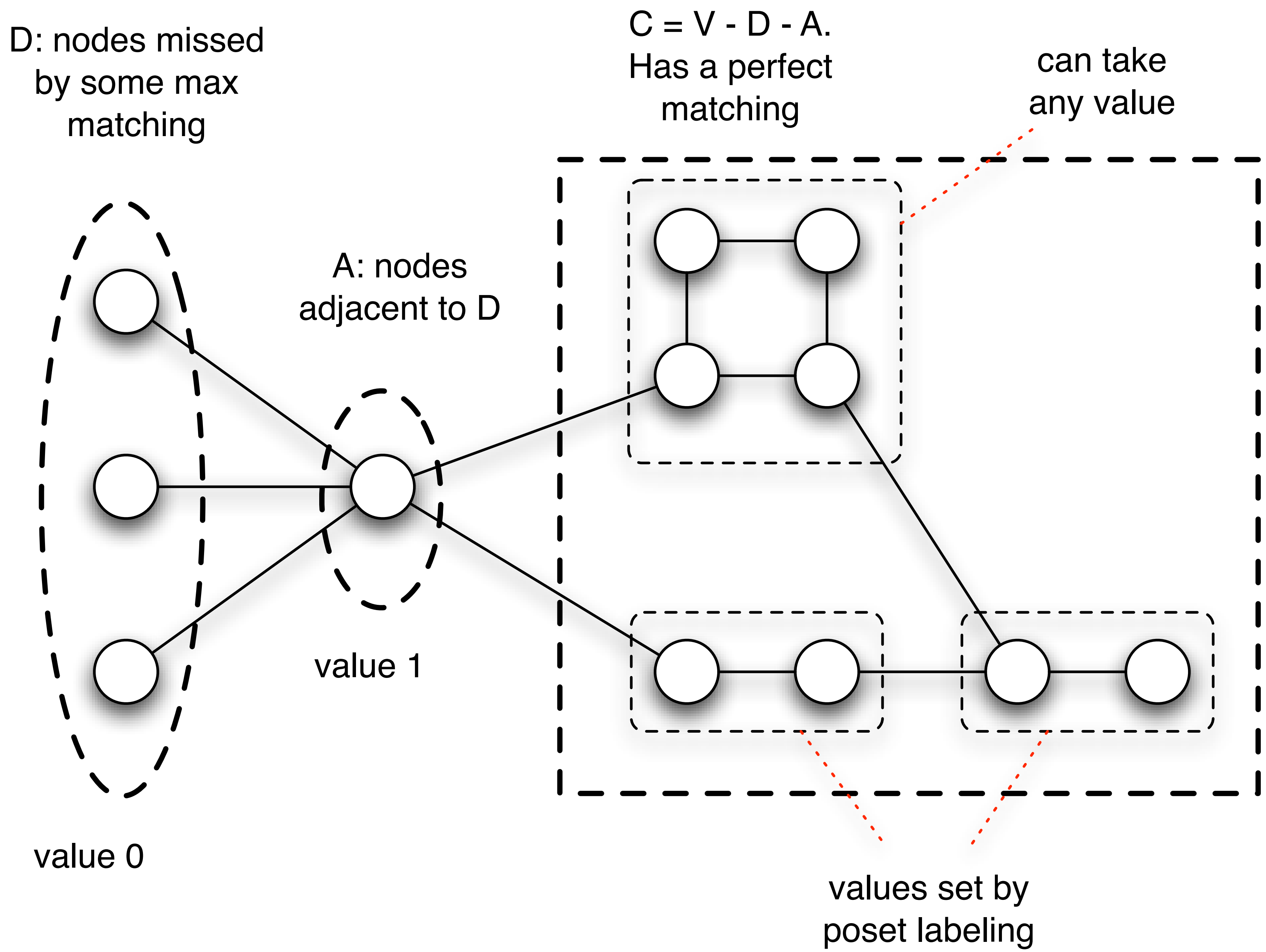
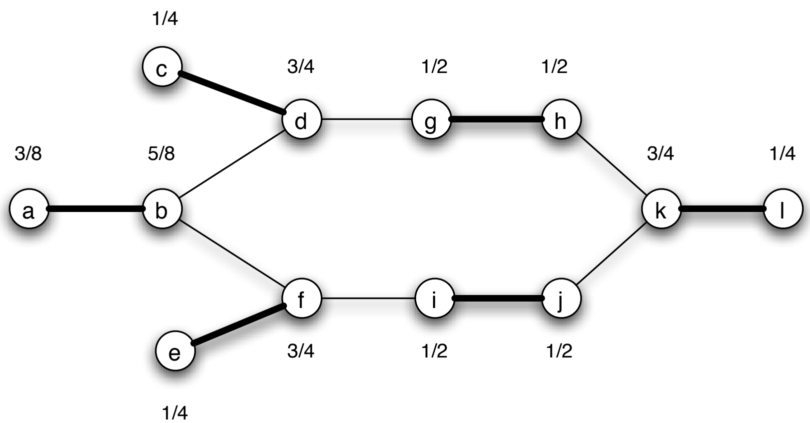
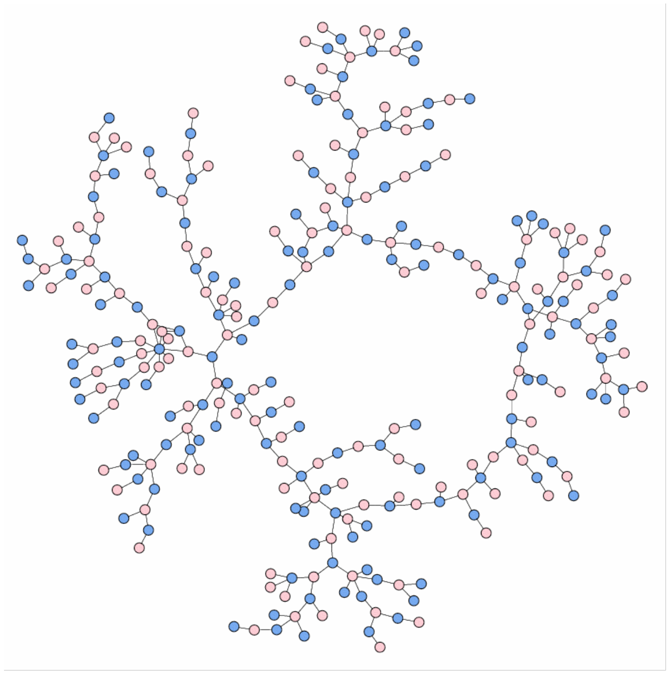

D: nodes missed
by some max
matching
A: nodes
adjacent to D
C = V - D - A.
Has a perfect
matching
can take
any value
value 1
value 0
values set by
poset labeling

Further Directions





- Can handle non-bipartite graphs using more complex decomposition and further structures that constrain values.
- Interesting connections to markets based on intermediation
  [Blume-Easley-Kleinberg-Tardos 2007]
- Realistic dynamics of negotation to yield balanced outcomes?
- What are the structural consequences when agents can strategically choose whom to link to in these settings?
  [Kranton-Minehart 2001, Even-Dar-Kearns-Suri 2007]