

## CS 271 (Spring 2013) — Assignment 6

Due: 03/14/2013

- (1) Read Sections 5.1, 5.4, and 5.5. In particular, read Section 5.1 a few times. (You can also already read Sections 5.2 and 5.3 — we'll get to those soon in class.) Again, I strongly recommend doing the reading and working through the examples in the book, as well as the book's advice for avoiding common errors.
- (2) The textbook likes to explain induction with two images: climbing a ladder, or knocking over a chain of dominos. Climbing a ladder — in my view — is actually not such a perfect explanation. Find at least two other natural processes in the real world in which you have a chain of very similar events which cause each other one by one, and which are started by one event. (Notice: there are many things, like viral spread of videos on Facebook, where you have one event, such as posting a video, leading to a chain of events. But there, the events branch out, i.e., one link to a video from one person can cause links from many others. Here, we are really looking for simple chains.)
- (3) Solve the following exercises from the textbook
  - (a) Section 5.1, Exercises 10, 14, 18, 30, 40, 50, 60, 74
  - (b) Section 5.4, Exercise 10, 22, 38, 40
- (4) [0 points]  
**Chocolate Problem (1 chocolate bar):** Suppose that you have a graph of  $n$  people. Each of them can be exactly one of  $k$  colors at any given time. (Think of these colors as representing technology or political preferences.) People like to be similar to their neighbors. Specifically, they will take turns updating their colors. When it is the turn of node  $i$ , he looks at all his neighbors, and chooses a plurality color among all their colors. (So suppose that that he had 2 red, 2 blue, 1 green, and 1 yellow neighbor. Then, he could choose red or blue, but not green or yellow.) If one of the plurality colors is the one he currently already has, then he sticks with it. (People are lazy.) Otherwise, he may choose any of the plurality ones. We repeatedly perform this update rule in some order of nodes, like  $1, 2, 3, \dots, n, 1, 2, 3, \dots, n, \dots$ . Prove that for every starting state of colors, this process will eventually reach a state where no person changes colors any more. (Sorry, no hints this time — this problem is really pretty easy, as far as chocolate problems go.)