

CS 271 (Spring 2013) — Assignment 10

Due: 04/30/2013

Homework 10 cannot be submitted late, as we will be giving out sample solutions at the end of class on April 30.

- (1) Reread the class policies on accessing online solutions to homework problems. It is not acceptable when about 20% of a class are caught cheating. Cases we have caught up to homework 7 will be reported as a first offense. If anything else comes up on this or subsequent homeworks and you have been caught cheating on earlier homeworks, then you will have two cases in front of Student Judicial Affairs. The consequences likely wouldn't be pretty.
- (2) Solve the following exercises from the textbook
 - (a) Section 7.4, Exercises 28, 30, 40, 44, 46
 - (b) End of Chapter 7, Exercises 4, 20, 22, 26, 28, 30

(3) [0 points]

Chocolate Problem (2 chocolate bars): We're going to look at "random graphs" again, but this time with a different distribution. We start with n points $\{0, 1, 2, 3, \dots, n-1\}$ arranged in a line, where each point/node has an edge to its two neighbors. (At the two ends, there is only one neighbor.) In addition, every node gets one more random outgoing link. For node i , its outgoing random link will go to node j with probability proportional to $1/|i-j|$. (The actual probability will be $\Theta(\frac{1}{\log(n)|i-j|})$, to ensure that the probabilities sum up to 1.) The random outgoing links of different nodes are independent.

The leftmost node is trying to get a package to the rightmost node as fast as possible. Each node can only pass on the package to one of his friends (i.e., left neighbor, right neighbor, or random outgoing friend). Each node follows the following simple rule: Always give the package to the friend closest to the right end of the line.

Prove that this gets the package to the right end of the line in $O((\log n)^2)$ steps in expectation.

If you find this question sufficiently interesting, you are also welcome to show that if we replaced the $\Theta(\frac{1}{\log(n)|i-j|})$ probability with $1/n$ — so each distance is equally likely for the random neighbor — then this wouldn't work. Nor would it work if we replaced it with $\Theta(1/|i-j|^2)$. These are not much more difficult than the one you were asked to prove.